An Instability in the Radiative Ionization of Atomic Hydrogen/Helium Gas [1]

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ABSTRACT

We show that the process of photoionizing a gas of atomic hydrogen and helium by line radiation whose energy is slightly above the helium single-ionization threshold is unstable if the helium fraction by number is less than approximately one half. However, in the two scenarios we consider here, based on the Decaying Dark Matter (DDM) model of cosmological reionization, there is no significant growth. In the first scenario we consider ionization and recombination to be approximately in equilibrium. This is relevant to high photon flux rates and early reionization, but in that case the heating is balanced by Compton cooling, which is very stabilizing. In the second scenario we ignore recombination. This is relevant to low photon flux rates or to the last stage of the reionization. In that case there is too little growth on a cosmological time scale to be significant.

Subject headings: cosmology: dark matter — cosmology: early universe — cosmology: large-scale structure of universe — hydrodynamics — instabilities

1.   Introduction

The hydrogen in the intergalactic medium (IGM) is highly ionized and has been at least since $z = 4.3$ (Steidel & Sargent 1987; Giallongo et al. 1994). Recent observations indicate that the helium in the IGM is mostly at least singly ionized, at least at large redshifts (Reimers & Vogel 1993; Jakobsen et al. 1994; Miralda-Escude 1993). Though the

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[1] Accepted by the Astrophysical Journal.

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IGM might be ionized mainly by quasars (Meiksin & Madau 1993; Madau 1992) or an early generation of stars (Fukugita & Kawasaki 1994), additional ionization by photons emitted by decaying dark matter (DDM) has many appealing features (Flannery & Press 1979; Sciama 1990; Sciama 1993; Sciama, Rees, & Scott 1991).

In particular, if a generation of neutrinos has a mass sufficient to close the universe, and if such a particle decays into a much lighter particle and a photon, the photon's energy will be near the ionization energies of hydrogen and helium. Specifically,

$$E_\gamma \approx \frac{1}{2}m_\nu = \frac{1}{2}91.5h^2 \,\text{eV} \tag{1}$$

where h is the Hubble constant in units of 100 km/s/Mpc. Thus, for $h > .55$, the decay photons would have sufficient energy to ionize hydrogen and for $h > .74$, they could (singly) ionize helium.

The decay photons would also provide a source of out-of-equilibrium energy which might lead to small-scale structure formation. Hogan (1992) proposed a mechanism of small-scale structure formation through the ionization of a pure-hydrogen plasma, but Bradford & Hogan (1994) showed that this mechanism does not work. We have continued to seek possible instabilities of the system, since a neutrino-dominated universe requires a non-gravitational mechanism for spawning structure on galactic scale and below and there is no fundamental reason why this system should be stable. The free energy per atom is sufficient for many e-foldings of instability growth ($E_\gamma \gg kT$), and the scale of instability can be comparable to proto-galactic structure. A hydrodynamical instability would mimic many of the desirable features found in the gas component of a Cold Dark Matter (CDM) model, as described for example in Miralda-Escude & Rees (1993).

1.1.   Helium-Ionization Instability Mechanism

Meiksin (1993) suggested that there is an instability if DDM photons have sufficient energy to ionize helium. The instability is thermal, i.e. it is based on low-density regions being preferentially heated. Low-density regions receive more heat because they have a larger ratio of neutral hydrogen to neutral helium and a hydrogen ionization deposits more energy in the plasma than a helium ionization (because there is a larger difference between the photon energy and the ionization threshold). The ratio of neutral hydrogen to neutral helium is larger in low density regions because they are more ionized and the higher ionization cross-section of helium (for photons just above its ionization threshold) ensures that it will absorb photons out of proportion to its abundance.

The reason that low-density regions are more ionized is different in the two scenarios considered below. When recombination equilibrium holds, it is due to the fact that recombination happens in proportion to the square of the density (because a positive ion and an electron need to find each other) while the ionization rate is proportional to the density (the rate of production of photons is assumed to be unrelated to the density). When recombination equilibrium doesn't hold, the low density regions are more ionized because there are more photons per atom.

Though any source of uniformly distributed photons will do for this instability, we concentrate on decaying neutrinos and parameterize the photon production rate by the neutrino lifetime in units of 10^{24} seconds, τ_{24}.

2. Perturbation Expansion and Matter Evolution Equations

We expand to first order in small perturbations about background values, and expand the perturbations in a Fourier series. We normalize some perturbations by the unperturbed value, e.g. for the number density, n,

$$n = n_0 \left(1 + \delta \ln n \; e^{i\vec{k}\bullet\vec{x}} \right) \tag{2}$$

The zero superscript denotes the unperturbed variable; it will be dropped in subsequent formulas for simplicity.

The evolution equation for the internal energy density, u, is

$$\frac{\partial}{\partial t} u = \gamma u \frac{\partial}{\partial t} \ln n + \mathcal{P} \tag{3}$$

where n is the number density, γ is the adiabatic expansion coefficient (5/3 for a monatomic gas), and $\mathcal{P}$ is the heating and cooling term.[3] This equation linearizes to

$$\mathcal{P}\delta \ln u + u \frac{\partial}{\partial t}\delta \ln u = \gamma u \frac{\partial}{\partial t}\delta \ln n + \delta\mathcal{P} + \mathcal{P}_u \delta \ln u \tag{4}$$

The relationship between density and pressure (or internal energy) perturbations is (see (Peebles 1980))

$$\frac{\partial}{\partial t}\frac{\partial}{\partial t}\delta \ln n + 2H\frac{\partial}{\partial t}\delta \ln n = -\frac{k^2}{a^2}\frac{v_s^2}{\gamma}\delta \ln u \tag{5}$$

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[3]Table 1 lists the symbols used in this article.

where k is the comoving wave number, H is the Hubble constant, a is the cosmic scale factor and is equal to $1/(1 + z)$, and v_s is the speed of sound (remember that the pressure is just $\gamma - 1$ times the internal energy density).

Here

$$\frac{v_s^2}{\gamma} = \frac{p_0}{\rho_0} = \frac{(1 + \chi)T}{m_{proton}(1 + 3\mu)} = \frac{(\gamma - 1)u}{nm_{proton}(1 + 3\mu)} \tag{6}$$

where μ is the fraction by number of helium nuclei and χ is the ratio of free electrons to total hydrogen and helium nuclei. Note that $\frac{m_{\text{proton}}(1+3\mu)}{(1+\chi)}$ is the average mass of a plasma particle (since $m_{\text{electron}} \ll m_{\text{proton}}$).

Combining eqs. 4 and 5 gives

$$\frac{\partial}{\partial t}\frac{\partial}{\partial t}\frac{\partial}{\partial t}\delta \ln n + a_2\frac{\partial}{\partial t}\frac{\partial}{\partial t}\delta \ln n + a_1\frac{\partial}{\partial t}\delta \ln n + \frac{v_s^2 k^2}{a^2 \gamma u}\delta \mathcal{P} = 0 \tag{7}$$

$$a_1 = 2H\left(\frac{\mathcal{P}}{u} - \mathcal{P}_u\right) + H^2 + v_s^2\frac{k^2}{a^2} \tag{8}$$

$$a_2 = \frac{\mathcal{P}}{u} - \mathcal{P}_u + 4H \tag{9}$$

Since a_1 and a_2 are positive (note that P_u is zero for ionization heating and is negative for Compton cooling), we will have an instability if $\frac{\delta \mathcal{P}}{\delta \ln n} < 0$, which we will show to hold for the ionization instability considered alone, though it doesn't hold when the ionization heating is balanced by Compton cooling.

Since a_1 and a_2 are positive, they lead only to damping of growth. We ignore them from now on, which can only be generous to the instability.

The analysis to this point has been general to any sort of heating. The form of $\delta \ln \mathcal{P}$ will be different for the two instability scenarios here, though in both $\mathcal{P}$ depends directly only on χ and the relationship between n and χ is used to get the variation of $\mathcal{P}$ with n. For the "recombination equilibrium" scenario, we will find $\delta \mathcal{P}$ in terms of $\delta \ln n$ and the equation will be third order; for the "no recombination" scenario we will have $\delta \mathcal{P}$ in terms of $\int \delta \ln n dt$ and the equation will be fourth order.

3. Scenario in which Recombination and Cooling are Important

3.1. Photon Energy Levels

At the heart of the ionization instability mechanism is the fact that a helium ionization deposits more energy than a hydrogen ionization. However, a significant part of that

difference is lost when the helium atom recombines, emitting photons which might then ionize a hydrogen atom.

A direct recombination to ground will yield a photon at the ionization energy of He, which we label E_1; if this photon ionizes a hydrogen atom, no energy will have been lost in the helium ionization. A recombination to an excited state of helium will emit a low-energy photon which can't ionize hydrogen, so its energy is lost. All such recombinations cascade (emitting other low-energy photons) to one of three states, which then decays directly to ground, usually emitting a photon which can ionize hydrogen (so we need to keep track of it). The $2\,^3P$ triplet state and the $2\,^1P$ singlet decay by emitting one photon at energy E_2 or E_3 respectively. Helium atoms which end up in the He $2\,^1S$ state can only decay to ground by the emission of two photons. We label the $2\,^1S$ state energy E_4 and the average energy of the two-photon-decay photons which can ionize hydrogen E_5 (see eq. (A2)). Finally, we label the the primary photon energy (the energy of the decay photons) E_0.

On average $p_1 = 0.554$ hydrogen ionizations result from each two-photon decay of the $2\,^1S$ state (eq. (A1)); all other helium recombinations lead (in the optically thick limit) to a hydrogen ionization.

See Osterbrock (1989) for more background on these radiative processes.

3.2.   Ionization and Recombination Equations

The heat energy absorbed will be proportional to the numbers of photons in each level times the absorption cross section for those photons:

$$
\begin{aligned}
\mathcal{P} = \ & n_{\gamma 0}[(E_0 - \mathrm{Ry})\sigma_{\mathrm{H}0}n_{\mathrm{H\,I}} + (E_0 - E_1)\sigma_{\mathrm{He}0}n_{\mathrm{He\,I}}] + n_{\gamma 1}(E_1 - \mathrm{Ry})\sigma_{\mathrm{H}1}n_{\mathrm{H\,I}} + \\
& n_{\gamma 2}(E_2 - \mathrm{Ry})\sigma_{\mathrm{H}2}n_{\mathrm{H\,I}} + n_{\gamma 3}(E_3 - \mathrm{Ry})\sigma_{\mathrm{H}3}n_{\mathrm{H\,I}} + n_{\gamma 5}(E_5 - \mathrm{Ry})\sigma_{\mathrm{H}5}n_{\mathrm{H\,I}}.
\end{aligned} \tag{10}
$$

$n_{\mathrm{H\,I}}$, $n_{\mathrm{H\,II}}$, $n_{\mathrm{He\,I}}$, $n_{\mathrm{He\,II}}$ are the number densities of neutral and singly ionized hydrogen and helium atoms. $n_{\gamma 0}$, $n_{\gamma 1}$, etc., are the number densities of photons with energy E_0, E_1, etc. Ry is one Rydberg, the hydrogen ionization energy. The sigmas are the absorption cross sections for the given substances with photons of the indicated energy, e.g. $\sigma_{\mathrm{He}1}$ is the cross section for the ionization of helium by photons with energy E_1.

The following equations express the assumption that the creation and destruction of photons at each level are in equilibrium:

$$
\begin{aligned}
\Re &= (\sigma_{\mathrm{He}0}n_{\mathrm{He\,I}} + \sigma_{\mathrm{H}0}n_{\mathrm{H\,I}})n_{\gamma 0} \\
\alpha_{\mathrm{He}1}n_{\mathrm{He\,II}}n_e &= (\sigma_{\mathrm{He}1}n_{\mathrm{He\,I}} + \sigma_{\mathrm{H}1}n_{\mathrm{H\,I}})n_{\gamma 1}
\end{aligned}
$$

$$\alpha_{\mathrm{He2\,^3P}} n_{\mathrm{He\,II}} n_e = \sigma_{\mathrm{H2}} n_{\mathrm{H\,I}} n_{\gamma 2}$$

$$\alpha_{\mathrm{He2\,^1P}} n_{\mathrm{He\,II}} n_e = \sigma_{\mathrm{H3}} n_{\mathrm{H\,I}} n_{\gamma 3}$$

$$\alpha_{\mathrm{He2\,^1S}} n_{\mathrm{He\,II}} n_e p_1 = \sigma_{\mathrm{H5}} n_{\mathrm{H\,I}} n_{\gamma 5} \tag{11}$$

$\Re$ is the rate of production of primary photons per unit volume (in the decaying neutrino model it is n_ν/τ_ν). The α's are the net recombination coefficients into various states, as defined in Table 1. Direct recombination of hydrogen to ground is ignored since (in the optically thick limit) the photon produced immediately ionizes another neutral hydrogen atom.

The following equations say that the creation and destruction of each species of ion are in equilibrium:

$$\alpha_{\mathrm{H2}} n_{\mathrm{H\,II}} n_e = (\sigma_{\mathrm{H0}} n_{\gamma 0} + \sigma_{\mathrm{H1}} n_{\gamma 1} + \sigma_{\mathrm{H2}} n_{\gamma 2} + \sigma_{\mathrm{H3}} n_{\gamma 3} + \sigma_{\mathrm{H5}} n_{\gamma 5}) n_{\mathrm{H\,I}}$$

$$\alpha_{\mathrm{He}} n_{\mathrm{He\,II}} n_e = (\sigma_{\mathrm{He0}} n_{\gamma 0} + \sigma_{\mathrm{He1}} n_{\gamma 1}) n_{\mathrm{He\,I}} \tag{12}$$

We also use the following relationships (from Osterbrock (1989)) between recombination coefficients:

$$\alpha_{\mathrm{He}} = \alpha_{\mathrm{He1}} + \alpha_{\mathrm{He2}}$$

$$\alpha_{\mathrm{He2\,^3P}} = \frac{3}{4}\alpha_{\mathrm{He2}}$$

$$\alpha_{\mathrm{He2\,^1P}} = \frac{2}{3}\left(\frac{1}{4}\alpha_{\mathrm{He2}}\right)$$

$$\alpha_{\mathrm{He2\,^1S}} = \frac{1}{3}\left(\frac{1}{4}\alpha_{\mathrm{He2}}\right) \tag{13}$$

3.3. Analysis

Combining eqs. (11), (12) and (13) gives the density and hydrogen ionization coefficient in terms of the helium ionization coefficient:

$$\chi_{\mathrm{H}} = \frac{(1-\mu)\sigma_{\mathrm{H1}}\alpha_{\mathrm{He}} + \frac{1}{12}(p_1 + 11)\sigma_{\mathrm{He1}}\mu(1-\chi_{\mathrm{He}})\alpha_{\mathrm{He2}}}{(1-\mu)(\sigma_{\mathrm{He1}}(1-\chi_{\mathrm{He}})\alpha_{\mathrm{H2}} + \sigma_{\mathrm{H1}}\chi_{\mathrm{He}}\alpha_{\mathrm{He}})}\chi_{\mathrm{He}} \tag{14}$$

$$n^2 = \frac{\sigma_{\mathrm{He1}}\Re(1-\chi_{\mathrm{He}})}{((1-\mu)\sigma_{\mathrm{H1}}(1-\chi_{\mathrm{H}})\alpha_{\mathrm{He}} + \sigma_{\mathrm{He1}}\mu(1-\chi_{\mathrm{He}})\alpha_{\mathrm{He2}})((1-\mu)\chi_{\mathrm{H}} + \mu\chi_{\mathrm{He}})\chi_{\mathrm{He}}} \tag{15}$$

Applying those relations to the formula for $\mathcal{P}$ (eq. (10)) gives

$$\mathcal{P} = \Re\left(\Delta E - \frac{\sigma_{\mathrm{He1}}\mu\alpha_{\mathrm{He2}}}{\sigma_{\mathrm{He1}}\mu\alpha_{\mathrm{He2}} + \sigma_{\mathrm{H1}}(1-\mu)\alpha_{\mathrm{He}}}E_{\mathrm{Loss}}\right)\frac{1 - A\chi_{\mathrm{He}}}{1 - B\chi_{\mathrm{He}}} \tag{16}$$

$$A = \frac{(\Delta E - E_{\text{Loss}})\sigma_{\text{He1}}\alpha_{\text{H2}} + \left(E_{\text{Loss}} - \frac{1}{12}(1 - p_1)\Delta E\right)\sigma_{\text{H1}}\alpha_{\text{He}}}{(\Delta E - E_{\text{Loss}})\sigma_{\text{He1}}\mu\alpha_{\text{He2}} + \Delta E\sigma_{\text{H1}}(1 - \mu)\alpha_{\text{He}}}\mu\frac{\alpha_{\text{He2}}}{\alpha_{\text{H2}}} \tag{17}$$

$$B = \frac{\sigma_{\text{He1}}\alpha_{\text{H2}} - \frac{1}{12}(1 - p_1)\sigma_{\text{H1}}\alpha_{\text{He}}}{\sigma_{\text{H1}}(1 - \mu)\alpha_{\text{He}} + \sigma_{\text{He1}}\mu\alpha_{\text{He2}}}\mu\frac{\alpha_{\text{He2}}}{\alpha_{\text{H2}}} \tag{18}$$

$$\tag{19}$$

ΔE is the difference between E_0 and a Rydberg and E_{Loss} is the difference in the energy deposited when a primary photon ionizes a hydrogen atom instead of a helium atom:

$$E_{\text{Loss}} = E_1 - \frac{1}{12}(E_5 p_1 + 2E_3 + 9E_2 + \text{Ry}(1 - p_1)) = 4.97\,\text{eV}. \tag{20}$$

To get positive feedback in the heating, we need the heating to go down with increasing density and hence to go up with increasing ionization. For that we need $A < B$ or

$$\sigma_{\text{He1}}(1 - \mu)\alpha_{\text{H2}} > \sigma_{\text{H1}}(1 - \mu)\alpha_{\text{He}} + \frac{1}{12}(p_1 + 11)\sigma_{\text{He1}}\mu\alpha_{\text{He2}} \tag{21}$$

which holds since $\mu \lesssim \frac{1}{2}$ (i.e. there is more hydrogen than helium), $\sigma_{\text{H1}} \ll \sigma_{\text{He1}}$ (i.e. the absorption cross-section for helium is larger at this energy), and $\frac{2}{3}\alpha_{\text{He}} \approx \alpha_{\text{He2}} \approx \alpha_{\text{H2}}$ (i.e. all the recombination coefficients are comparable).

Filling in the numeric values given in Table 1 and taking $E_0 = E_1$ (that is, ionization photons right at the helium ionization threshold, which is most generous to the instability), we get $\mathcal{P}$ and n^2 in terms of the variable χ_{He} and the parameter τ_{24}:

$$\mathcal{P} = 1.12 \times 10^{-21}\frac{\text{eV}\,z'^3}{\text{cm}^3 s}\frac{1 - 0.180\chi_{\text{He}}}{1 - 0.246\chi_{\text{He}}} \tag{22}$$

$$n^2 = 2.29 \times 10^{-9}\frac{1}{\tau_{24}\text{cm}^6}\frac{(1 - 0.736\chi_{\text{He}})^2}{(1 - 0.243\chi_{\text{He}})(1 - 0.326\chi_{\text{He}})\chi_{\text{He}}^2} \tag{23}$$

where $z' = 1 + z$. When varied these give

$$\delta\mathcal{P} = 0.0662\,\mathcal{P}\frac{1}{(1 - 0.180\chi_{\text{He}})(1 - 0.246\chi_{\text{He}})}\delta\chi_{\text{He}} \tag{24}$$

(note the small coefficient in this equation, indicating the weakness of this instability) and

$$\delta\chi_{\text{He}} = \frac{\delta n}{n}\frac{(1 - 0.246\chi_{\text{He}})^2(1 - 0.351\chi_{\text{He}})^2(1 - 0.736\chi_{\text{He}})^2\chi_{\text{He}}}{(1 - 0.600\chi_{\text{He}} + 0.266\chi_{\text{He}}^2)(1 - 0.243\chi_{\text{He}})(1 - 0.295\chi_{\text{He}})(1 - 0.326\chi_{\text{He}})(1 - 0.771\chi_{\text{He}})} \tag{25}$$

Defining the coefficient multiplying $\delta\ln n$ in eq. 7 as

$$a_0 = \frac{v_s^2 k^2}{a^2\gamma u}\mathcal{P}_n \tag{26}$$

and using $a^{-2} = (H/H_0)^3$ we get

$$a_0 = -\frac{2.56(1 - 0.35\chi_{\mathrm{He}})^2(1 - 0.74\chi_{\mathrm{He}})^2\chi_{\mathrm{He}}(\mathrm{Mpc}\,k)^2H^3}{(1 - 0.60\chi_{\mathrm{He}} + 0.23\chi_{\mathrm{He}}{}^2)(1 - 0.24\chi_{\mathrm{He}})(1 - 0.29\chi_{\mathrm{He}})(1 - 0.33\chi_{\mathrm{He}})(1 - 0.77\chi_{\mathrm{He}})z'^{5/2}\tau_{24}} \tag{27}$$

We can use eq. 23 and the known background value for the density, $n = 1.04 \times 10^{-7}\frac{z'^3}{cm^3}$ (this assumes $\eta = 3 \times 10^{10}$) to get the background ionization for a given redshift:

$$\tau_{24}z'^3 = 4.06 \times 10^5 \frac{(1 - 0.78\chi_{\mathrm{He}})^2}{\chi_{\mathrm{He}}{}^2(1 - 0.27\chi_{\mathrm{He}})(1 - 0.37\chi_{\mathrm{He}})} \tag{28}$$

Using this equation to eliminate the redshift gives

$$a_0 = -\frac{9.30 \times 10^{-5}(1 - 0.35\chi_{\mathrm{He}})^2(1 - 0.74\chi_{\mathrm{He}})^{1/3}\chi_{\mathrm{He}}^{8/3}(\mathrm{Mpc}\,k)^2H^3}{(1 - 0.60\chi_{\mathrm{He}} + 0.23\chi_{\mathrm{He}}{}^2)(1 - 0.24\chi_{\mathrm{He}})^{1/6}(1 - 0.29\chi_{\mathrm{He}})(1 - 0.33\chi_{\mathrm{He}})^{1/6}(1 - 0.77\chi_{\mathrm{He}})\tau_{24}^{1/6}} \tag{29}$$

This is largest (in magnitude) at $\chi_{\mathrm{He}} = 1$, where it is

$$a_0 = -2.79 \times 10^{-4}\frac{(\mathrm{Mpc}\,k)^2H^3}{\tau_{24}^{1/6}} \tag{30}$$

For a sufficiently small time, a_0 will be effectively constant and the growth rate will be

$$\omega_g = \sqrt[3]{-a_0} \tag{31}$$

We will show that there is no growth for any such small time interval, so there will be no growth for any larger time interval.

3.4. Range of the Instability

The minimum scale for this instability is the point where the assumption that the medium is optically thick breaks down; it is the optical path length (which should be the minimum of the lengths for a helium or a hydrogen ionization, but for convenience we take it to be the length for a helium ionization):

$$l_{\min} = \frac{1}{\sigma_{\mathrm{He}}(1 - \chi_{\mathrm{He}})n_{\mathrm{He}}} \tag{32}$$

or (still with $E_0 = E_1$)

$$l_{\min}z' = 5.46\mathrm{Mpc}\frac{1}{(1 - \chi_{\mathrm{He}})z'^2} \tag{33}$$

or in terms of wave number,

$$k_{\max} = \frac{2\pi}{l_{\min}z'} = 1.15 \mathrm{Mpc}^{-1}(1 - \chi_{\mathrm{He}})z'^2. \tag{34}$$

The maximum scale comes from requiring that the growth rate be larger than the expansion rate: $\omega_g > H$. Taking ω_g from eq. 31 gives

$$k_{\min} = 104. \, \mathrm{Mpc}^{-1}\tau_{24}^{1/12} \times$$
$$\sqrt{\frac{(1 - 0.60\chi_{\mathrm{He}} + 0.27\chi_{\mathrm{He}}{}^2)(1 - 0.24\chi_{\mathrm{He}})^{1/6}(1 - 0.30\chi_{\mathrm{He}})(1 - 0.33\chi_{\mathrm{He}})^{1/6}(1 - 0.77\chi_{\mathrm{He}})}{(1 - 0.35\chi_{\mathrm{He}})^2(1 - 0.74\chi_{\mathrm{He}})^{1/3}\chi_{\mathrm{He}}^{8/3}}} \tag{35}$$

where the ionization is related to the redshift through eq. 28.

Requiring that the instability have a non-zero range of scales, or

$$k_{\min} < k_{\max} \tag{36}$$

gives a limit on χ_{He} or equivalently z' for a particular τ_{24}. Figure 1 shows the lower bound on z' as a function of τ_{24}. The lowest z' at which there is a finite range for the instability is $z' = 21.0$ (for $\tau_{24} = 20.7$ and $\chi_{\mathrm{He}} = 0.665$).

An additional constraint is that the assumption of ionization equilibrium be valid. Define the ionization rate per helium ion as $\omega_{\mathrm{Rec}} = \alpha_{\mathrm{He}}n_e = \alpha_{\mathrm{He}}\chi\rho$. For these parameters $\omega_{\mathrm{Rec}} = 1.01H$ so equilibrium still (marginally) holds.

3.5. Compton Cooling

In the post-recombination, pre-galactic universe of our model, the dominant cooling mechanism is Compton scattering off the background radiation (since in the DDM model there will always be a significant ionized fraction).

The power lost to Compton cooling is

$$\mathcal{L} = 4aT_\gamma^4\frac{\sigma_{\mathrm{Thompson}}}{cm_{\mathrm{electron}}}n_e(T - T_\gamma) = 4aT_\gamma^4\frac{\sigma_{\mathrm{Thompson}}}{cm_{\mathrm{electron}}}(\frac{\chi u}{1 + \chi} - n\chi T_\gamma) \tag{37}$$

which consists of a cooling term proportional to $u\frac{\chi}{1+\chi}$ and a heating term proportional to $n\chi$. The cooling term increases with ionization and thus opposes any instability. The heating term is approximately constant with density or ionization (since χ varies approximately as $1/n$) but is also (weakly) stabilizing.

Since Compton cooling depends more directly on the ionization ($\mathcal{L}_\chi \approx \mathcal{L}$), its stabilizing effect dominates if the cooling is comparable in magnitude to the photo-ionization heating. With a simple model for the temperature[4], we calculate that at the lowest redshift possible, $z' = 21.0$, the ratio of the cooling density dependence to the heating density dependence is

$$\frac{\mathcal{L}_n}{\mathcal{P}_n} < -2.68. \tag{38}$$

Figure 2 shows $\frac{\mathcal{L}_n}{\mathcal{P}_n}$ as a function of τ_{24} at the minimum z' using the same thermal model. It's smallest magnitude is

$$\frac{\mathcal{L}_n}{\mathcal{P}_n} < -2.00 \tag{39}$$

at $z' = 24.3$, $\tau_{24} = 55.7$, and $\chi_{\mathrm{He}} = 0.409$.[5]

Thus, the stabilization of Compton cooling will always dominate the destabilizing effects of the heating.

4. Scenario in which Recombination is Ignored

At redshifts or ionization rates which are low enough that Compton cooling is not significant, recombination is also less important. We model this regime as having no recombination, which strengthens the instability, but reduces the time over which it can operate.

4.1. Ionization Equations

We solve for the ionization by still assuming photon production to be in equilibrium with ionization:

$$\Re = (\sigma_{\mathrm{He}0} n_{\mathrm{He\,I}} + \sigma_{\mathrm{H}0} n_{\mathrm{H\,I}}) n_{\gamma 0} \tag{40}$$

but ionization is not in equilibrium with recombination:

$$\frac{d\chi_{\mathrm{He}}}{dt} = n_{\gamma 0} \sigma_{\mathrm{He}0} (1 - \chi_{\mathrm{He}}) \tag{41}$$

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[4] To guarantee that the model temperature was a lower bound, we took the ionization in $\mathcal{L}$ to be constant at 1 and the ionization in $\mathcal{P}$ to be constant at 0 and we approximated the resulting integral for u in a way that gives about half the actual value for the difference in the matter temperature and the photon temperature.

[5] For these parameters, $\omega_{\mathrm{Rec}} = 0.612 H$, so the assumption of ionization equilibrium is invalid. Enforcing that assumption would require an even less favorable ratio.

$$\frac{d\chi_{\mathrm{H}}}{dt} = n_{\gamma 0}\sigma_{\mathrm{H0}}(1 - \chi_{\mathrm{H}}). \tag{42}$$

Taking the ratio of eqs. (41) and (42) we get

$$\frac{d\chi_{\mathrm{H}}}{d\chi_{\mathrm{He}}} = \frac{\sigma_{\mathrm{H0}}}{\sigma_{\mathrm{He0}}}\frac{1 - \chi_{\mathrm{H}}}{1 - \chi_{\mathrm{He}}} \tag{43}$$

or

$$1 - \chi_{\mathrm{H}} = c_1(1 - \chi_{\mathrm{He}})^{\frac{\sigma_{\mathrm{H0}}}{\sigma_{\mathrm{He0}}}} \tag{44}$$

where c_1 is determined by the initial conditions. This is an exact equation which holds for the background ionization and which can be varied to yield a relationship between the hydrogen and helium perturbations:

$$\delta\chi_{\mathrm{He}} = c_1\frac{\sigma_{\mathrm{H0}}}{\sigma_{\mathrm{He0}}}(1 - \chi_{\mathrm{He}})^{\frac{\sigma_{\mathrm{H0}}}{\sigma_{\mathrm{He0}}} - 1}\delta\chi_{\mathrm{H}} \tag{45}$$

From eqs. (41), (44), and (40) we get

$$(1 - \mu)(1 - \chi_{\mathrm{H}}) + \mu(1 - \chi_{\mathrm{He}}) = -\int\frac{\Re}{n}dt \tag{46}$$

Note that $\frac{\Re}{n}$ is a constant in the unperturbed background, so for background quantities:

$$(1 - \mu)(1 - \chi_{\mathrm{H}}) + \mu(1 - \chi_{\mathrm{He}}) = -\frac{\Re}{n}t \tag{47}$$

which gives the ionization as an implicit function of time or (as we shall use it) gives the time as a function of the background ionization.

Using the following formula (valid for a flat, matter-dominated universe) we can express the redshift in terms of time and thus ionization.

$$t = \frac{2}{3}\frac{1}{H_0}(z')^{-\frac{3}{2}} \tag{48}$$

Varying eq. 46 gives

$$(1 - \mu)\delta\chi_{\mathrm{H}} + \mu\delta\chi_{\mathrm{He}} = -\frac{\Re}{n}\int\delta\ln n\, dt \tag{49}$$

which relates the variation of the density to the variation of the ionization.

4.2. The Variation of Heating with Density

The heating function, $\mathcal{P}$, is simpler than in the first scenario because there are no photons at reprocessed energy levels:

$$\mathcal{P} = n_{\gamma 0}[(E_0 - \mathrm{Ry})\sigma_{\mathrm{H}0}n_{\mathrm{H\,I}} + (E_0 - E_1)\sigma_{\mathrm{He}0}n_{\mathrm{He\,I}}]. \tag{50}$$

Using eq. 40 to eliminate $n_{\gamma 0}$ gives

$$\mathcal{P} = \mathcal{R}\left[(E_0 - \mathrm{Ry}) - (E_1 - \mathrm{Ry})\frac{1}{1 + \frac{1-\mu}{\mu}\frac{\sigma_{\mathrm{H}0}}{\sigma_{\mathrm{He}0}}\frac{1-\chi_{\mathrm{H}}}{1-\chi_{\mathrm{He}}}}\right] \tag{51}$$

As in the equilibrium case, the heating function depends on the density only through the ionization. Varying $\mathcal{P}$ with respect to the ionization and using the relationship between ionization and density variations (eq. 49) gives $\delta\mathcal{P}_n$ which substituted into eq. 7 gives

$$\frac{\partial}{\partial t}\frac{\partial}{\partial t}\frac{\partial}{\partial t}\delta\ln n + a_2\frac{\partial}{\partial t}\frac{\partial}{\partial t}\delta\ln n + a_1\frac{\partial}{\partial t}\delta\ln n - a_0\int\delta\ln n\,dt \tag{52}$$

where

$$a_0 = \frac{(\gamma-1)(1-\mu)}{(3\mu+1)\mu^2}\frac{\sigma_{\mathrm{H}0}}{\sigma_{\mathrm{He}0}}\left(1 - \frac{\sigma_{\mathrm{H}0}}{\sigma_{\mathrm{He}0}}\right)\frac{E_1 - \mathrm{Ry}}{m_{\mathrm{proton}}}\frac{1-\chi_{\mathrm{H}}}{(1-\chi_{\mathrm{He}})^2}\frac{1}{\left(1 + \frac{1-\mu}{\mu}\frac{\sigma_{\mathrm{H}0}}{\sigma_{\mathrm{He}0}}\frac{1-\chi_{\mathrm{H}}}{1-\chi_{\mathrm{He}}}\right)^3}\frac{\mathcal{R}^2}{n}k^2 z'^2 \tag{53}$$

and a_1 and a_2 are given by eqs. 8 and 9.

4.3. Bounds to the Growth

In this scenario an instability can grow, though the total growth is less than one e-folding so it is insignificant.

The time available for growth is bounded by the neutral atoms being used up, since there is no recombination. The starting ionization will be greater than zero since some ionization will occur while Compton cooling damps any growth. We take χ_{Orig} to be the recombination-equilibrium helium ionization (eqs. 15 and 14) at the red-shift where the damping from the Compton cooling just becomes weaker than the instability growth. This is generous to the model in three ways: 1) assuming recombination equilibrium always over estimates the remaining neutral fraction; 2) the Compton cooling is calculated for matter at the photon background temperature, where it is lowest (the net cooling will be zero at this temperature, but there is a non-zero derivative of the cooling with respect to helium

ionization); and 3) the growth will actually be zero at that point, not its full value without Compton cooling. Figure 3 shows $\chi_{\rm Orig}$ as a function of τ_{24}. $z'_{\rm Orig}$ and the original $\chi_{\rm H}$ can be obtained from $\chi_{\rm Orig}$ and τ_{24} using eqs. 28 and 14. Those parameters then determine c_1 from eq. 44 and the constant of integration in eq. 49.

The largest ionization, $\chi_{\rm Final}$ is the value of the helium ionization at which the mean-free-path exceeds the scale under consideration (i.e. it depends on k). It is determined for a given k from eq. 34 (given the redshift/ionization relationship of eq. 49).

To determine the total growth, we broke the evolution down in steps with a_0 taken to be constant at its maximum value in each step (the maximum is achieved for the smaller ionization at the start of the step). For constant a_0 (and ignoring a_2 and a_3, which only damp any growth) the growing solution is $\delta \ln n \propto \exp(\sqrt[4]{a_0}t)$. We took steps in $\chi_{\rm He}$ instead of time since we have time and redshift as a function of $\chi_{\rm He}$ and not vice-versa. Thus the growth in $\delta \ln n$ is bounded by $\exp(\sum_{i=0}^{N-1}(t(\chi_{i+1}) - t(\chi_i))\sqrt[4]{a_0(\chi_i)})$ for any N (with a tighter bound for a larger N) with $\chi_i = \chi_{\rm Orig} + \frac{i}{N}(\chi_{\rm Final} - \chi_{\rm Orig})$.

Figure 4 shows the log of the total growth (for $N = 40$) as a function of $\chi_{\rm Orig}$ and $\chi_{\rm Final}$. It's largest value is $\exp(.83)$ at $\chi_{\rm Orig} = .44$ and $\chi_{\rm Final} = .79$ (for these parameters the model is actually invalid because recombination is large, but it shows that within the space of validity the growth will be even less).

5. Conclusions

Though there is an interesting instability involving helium ionization by line radiation, it doesn't seem to have a cosmological application in the DDM scenario. As other candidate radiation sources presume the existence of small-scale structure, we conclude that this class of instability is unlikely to play a role in the initial formation of structure from smooth cosmic gas.

6. Acknowledgments

A.   Calculation of Two-Photon Decay Parameters

The He $2\,^1S$ state decays by the emission of two photons whose energy sums to E_4. The strength of the transition which emits one of the photons in the range ν to $\nu + d\nu$ is $A(\nu)d\nu$. We did an empirical fit to the average of the two tables of values for A given in Jacobs (1971). Figure 5 shows the fit and compares it to Jacobs's values.

The total strength for emission of two photons is $A_{\text{Tot}} \equiv \int_0^{E_4} A(\nu)d\nu$. The total strength for the events in which a particular one of the photons is energetic enough to ionize hydrogen is $A_{\text{Ry}} \equiv \int_{\text{Ry}}^{E_4} A(\nu)d\nu$. Since we have a hydrogen ionization if either photon is energetic enough, the proportion of decays which ionize hydrogen is[6]

$$p_1 = 2\frac{A_{\text{Ry}}}{A_{\text{Tot}}} = 0.554 \tag{A1}$$

Weighting the strength of decay by the energy of the decay gives the average energy of emission, here taken over only those photons which can ionize hydrogen:

$$E_5 = \frac{\int_{\text{Ry}}^{E_4} \nu A(\nu)d\nu}{A_{\text{Ry}}} = 16.1\text{eV} \tag{A2}$$

Since the absorption cross section depends on frequency, it should also be calculated as a weighted average:

$$\sigma_{\text{H5}} = \frac{\int_{\text{Ry}}^{E_4} \sigma_{\text{H}}(\nu)A(\nu)d\nu}{A_{\text{Ry}}} \tag{A3}$$

$$= 1.050\sigma_{\text{H}}(E_5) \tag{A4}$$

so the weighting gives a 5% correction compared to using the absorption cross section for line radiation at energy E_5.

B.   Ignored Effects

Recombinational cooling, or the thermal energy lost when two particles which both have average kinetic energy T combine to one particle with average energy T, is insignificant. Since each ionization results in a recombination in the first scenario, the ratio

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[6]This differs slightly from .56 given in Osterbrock (1989), perhaps because his value was not updated when he updated E_4 from 20.7 to 20.6.

of recombination cooling to ionization heating is roughly the ratio of the average energy deposited by an ionization to the temperature, or about $T/6.5\text{eV}$. Compton cooling keeps the matter temperature comparable to the radiation temperature, which is much less than 1 eV. For the second scenario there is no recombination, and thus no recombinational cooling. Note that no collisional ionizations happen at the low temperatures considered here.

Collisional Transitions between helium levels are insignificant at the cosmological densities which are of primary concern in this analysis. The critical density given in Osterbrock (1989) above which one must consider collisional transitions is $\sim 5000 cm^{-3}$, much larger than cosmological densities of $\sim 10^{-7} z'^3 cm^{-3} < 100 cm^{-3}$ for $z' < 1000$.

The magnitude of the **photon redshift** over one mean-free-path (for, e.g., an E_1 photon to ionize helium) is

$$\Delta z = 1.8 \times 10^{-3} h \frac{1}{z'^{\frac{3}{2}}(1 - \chi_{\text{He}})} \tag{B1}$$

An E_1 photon will be redshifted by more than one helium thermal Doppler over one mean-free-path for $z \lesssim 100$; this effectively reduces the absorption cross section of helium, which weakens the instability. The redshift over the mean-free-path of a one Rydberg photon is greater than a hydrogen thermal Doppler width for all $z \lesssim 50$; for smaller redshifts than that, some hydrogen recombinations direct to ground will be allowed, which slightly strengthens the instability, as can be seen by considering eq. (21) with increased α_{H2}.

A **distribution of initial photon energies** will result in reduced growth over a line spectrum with an energy just above the Helium ionization threshold simply because any photons much above the threshold are less effective and any photons below the threshold actually oppose the instability.

Lyman-alpha trapping will occur, since for $z \gtrsim 40$, the redshifting in one mean-free-path is insufficient to bring a Lyman-α photon more than one thermal Doppler width away from the line center, but because the time photons spend traveling between encounters is much larger than the time a hydrogen atom spends in the ^2P state, only approximately 10^{-7} of the neutral hydrogen atoms are excited, and this does not affect the instability.

Photon diffusion (relaxing the assumption that the medium is optically thick) opposes an instability, since the more-ionized/less-dense regions are more optically thin and would thus lose photons (and therefor energy) to the less-ionized/more-dense regions.

Thermal conduction opposes any instability and its effects are insignificant on the large scales considered here.

Collisional ionization goes up with temperature, but it goes up with density squared

so it would also act against an instability. Its rate is insignificant at the temperatures and densities of the post-recombination universe.

The **temperature dependence of the recombination coefficients** does not have a large effect because only the ratios of recombination coefficients enter the formulas, and the recombination coefficients for helium and hydrogen both depend on temperature to roughly the same power (3/4 for $T \approx 10000K$).

Any **delay in thermalization** has no effect, since Compton cooling and pressure are both linear in the sum of the energy of the particles, so the distribution of the energy does not matter.

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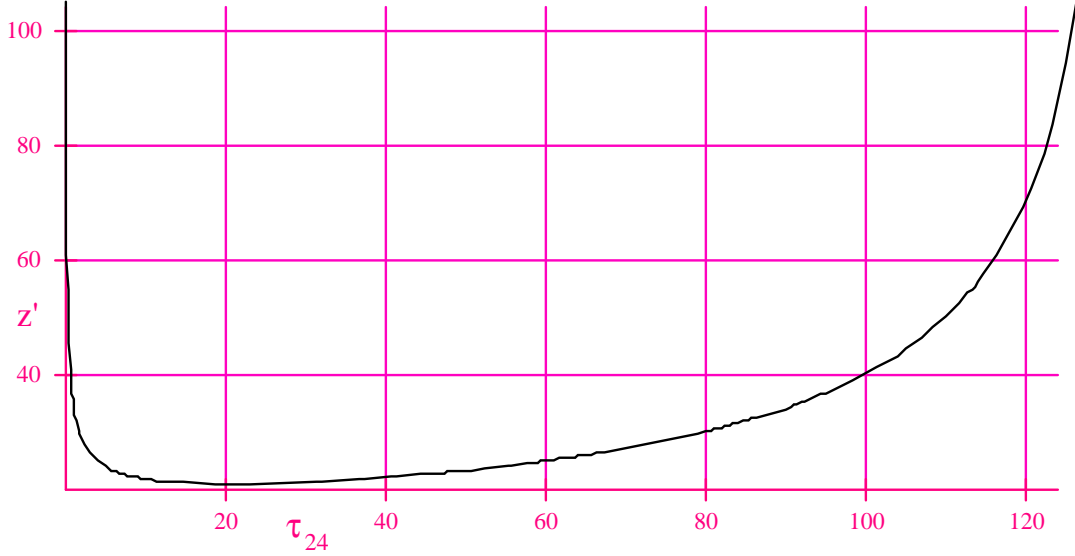
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Fig. 1.— Minimum $z' = z + 1$ vs $\tau_{24} = \tau_\nu/10^{24}$. For small τ_{24} (large photon production rate), the minimum redshift is large because the large equilibrium ionization leads to a large mean free path. For large τ_{24} the minimum redshift is large because the low ionization leads to low instability growth.
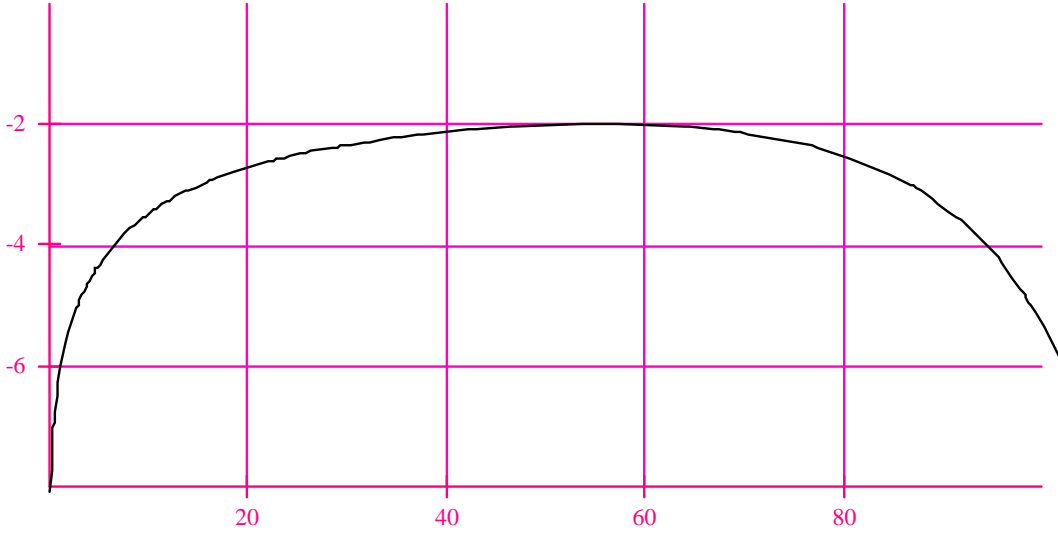

Fig. 2.— $\frac{\mathcal{L}_n}{\mathcal{P}_n}$ vs τ_{24}. The curve follows the inverse of the minimum z' curve (since Compton cooling is much more effective at large z), with a shift towards larger τ_{24} (where the ionization and thus the cooling is less).
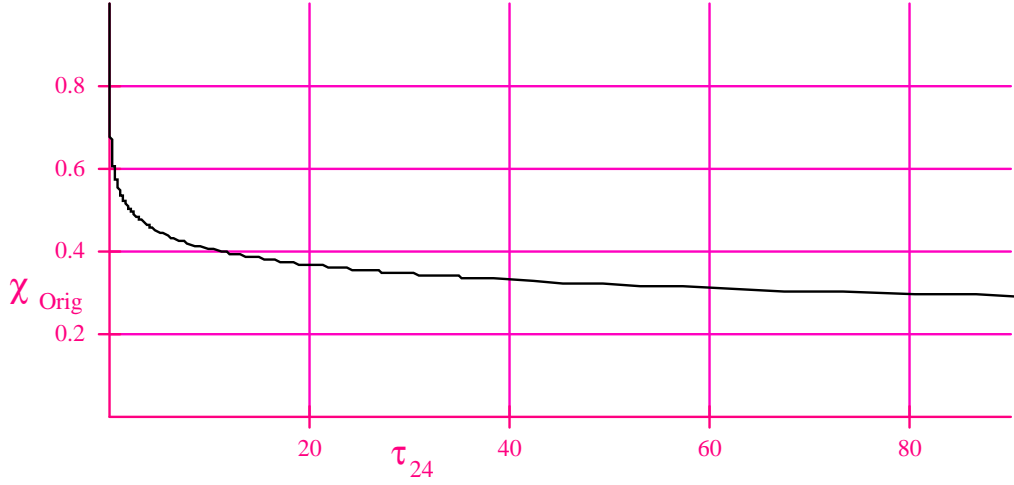
Fig. 3.— Starting χ_{He} for no-recombination conditions vs τ_{24}. This is the recombination-equilibrium ionization where the Compton cooling ceases to dominate the heating. The steep slope near $\tau_{24} = 0$ reflects the fact that the heating approaches zero as χ_{He} goes to one, so there needs to be a high photon production rate to compensate.
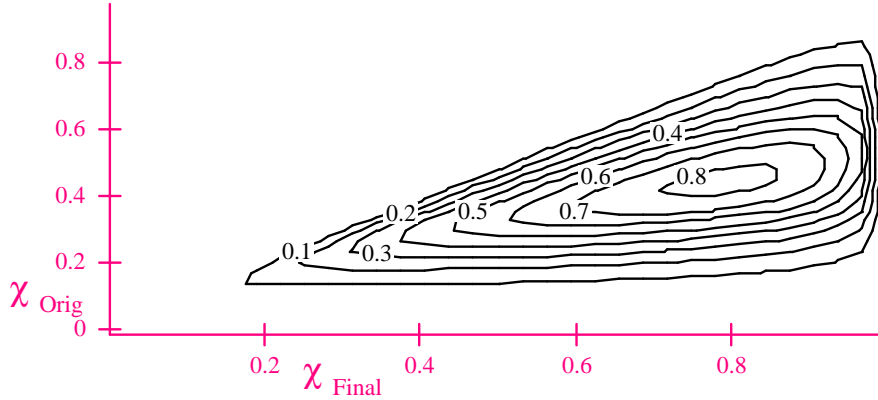

Fig. 4.— Logarithm of the total growth factor vs the initial and final ionization for no-recombination growth. Since ionization increases with time, we clearly need the final ionization to be larger than the initial ionization, with larger values of their difference giving more time for growth. However, very small values for the original ionization correspond to low photon production rates, which give small overall growth. Large values for the final ionization correspond to large spatial scales, where the growth is also reduced.
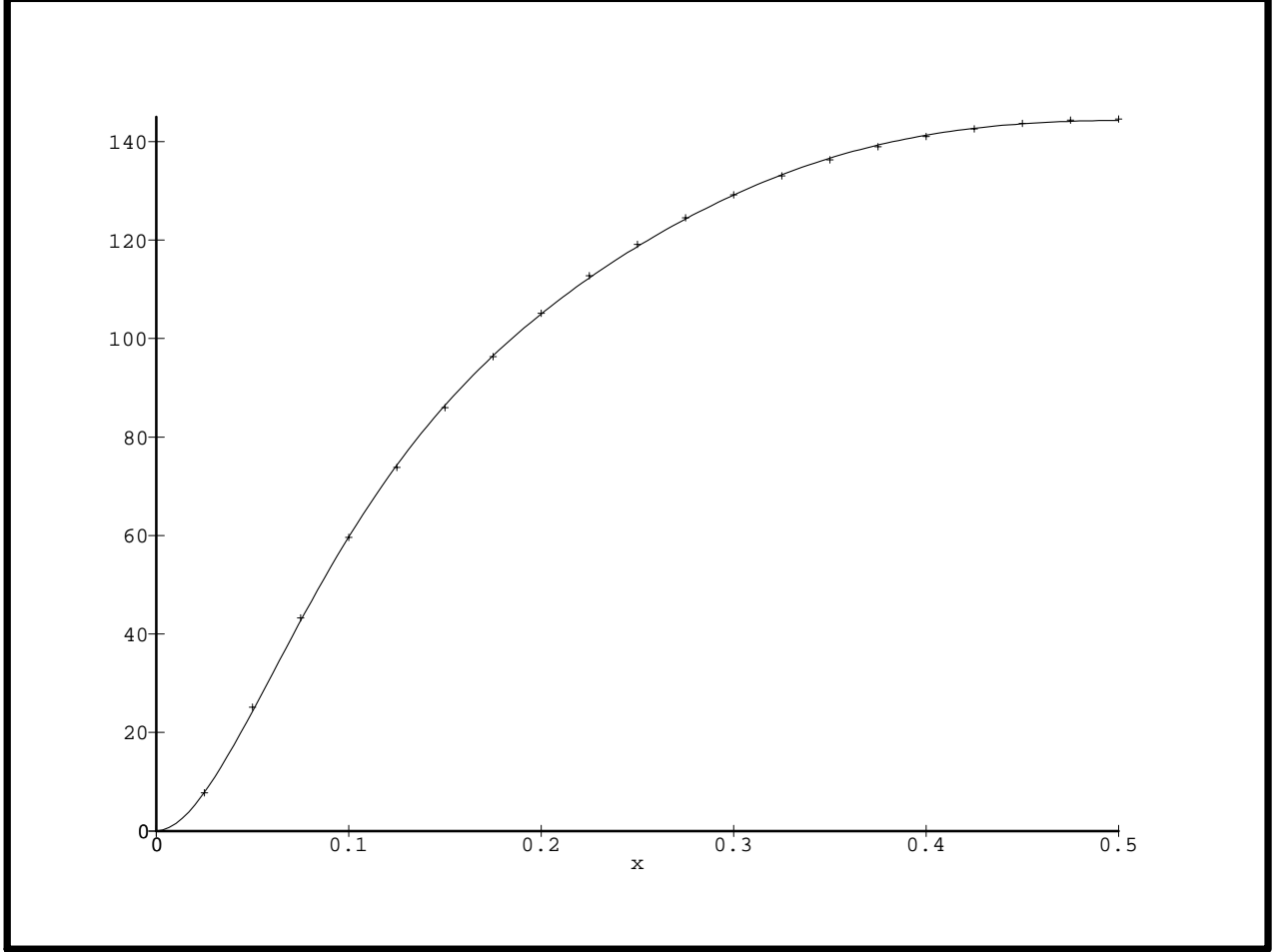
Fig. 5.— He $2\,^1S$ two-photon decay rate versus energy $(x = \nu/E_4)$. The fitting formula is $A(x) = -0.98p(2, x) + 2.83p(3, x) - 4.26p(4, x) + 2.15p(5, x)$, where $p(j, x) = 300((2x-1)^{2j} - 1 - j((2x-1)^2 - 1))$ (this form is chosen so that each term is symmetric around x=0.5 and is zero and has zero slope at $x = 0$ and $x = 1$). The RMS error of the fit is 0.34.

Table 1. Symbols

Symbol/Value	Explanation
h	The Hubble Parameter in units of 100 km/s/Mpc.
$\omega_g = -i\omega$	Instability growth rate.
$\vec{k}$	Wave vector in Fourier decomposition.
$k \equiv |k|$	Wave number (inverse of size scale).
n	Total spatial density of hydrogen plus helium nuclei (neutral and ionized).
$u = \frac{1}{\gamma-1}(1+\chi)nT_m$	Internal thermal energy of the gas.
$\gamma = 5/3$	Adiabatic expansion coefficient.
$\mathcal{P}$	Heat input per unit volume.
v_s	The unperturbed speed of sound.
$\mu = \frac{Y_p}{4-3Y_p} \approx 0.073$	Fraction by number of helium atoms ($Y_p \approx .24$ is used here).
$\omega_u = \mathcal{P}_u$	Miscelaeous rates.
$\omega_k = v_s k$	
$\omega_n = -\frac{\mathcal{P}_n n}{\gamma u}$	
$p_1 = 0.554$	Number of photons produced in a two-photon decay of the He $2\,^1S$ state which can ionize H.
$n_{\gamma 0}, n_{\gamma 1}$, etc.	Spatial density of photons with energy E_0, E_1, etc.
$n_{\mathrm{He\,II}} = n\mu\chi_{\mathrm{He}}$	Number density of ionized helium.
$n_{\mathrm{He\,I}} = n\mu(1-\chi_{\mathrm{He}})$	Number density of neutral helium.
$n_{\mathrm{H\,II}} = n(1-\mu)\chi_{\mathrm{H}}$	Number density of ionized hydrogen.
$n_{\mathrm{H\,I}} = n(1-\mu)(1-\chi_{\mathrm{H}})$	Number density of neutral hydrogen.
$n_e = n_{\mathrm{He\,II}} + n_{\mathrm{H\,II}} = n\chi$	Number density of electrons.
χ_{He}	Fractional ionization of helium.
χ_{H}	Fractional ionization of hydrogen.
$\chi = \mu\chi_{\mathrm{He}} + (1-\mu)\chi_{\mathrm{H}}$	Fractional ionization of all species combined.
$\sigma_{\mathrm{H}}(E) = 6.30\times10^{-18}\times$ $x^{-4}\frac{e^{4(1-y\,\mathrm{arccot}(y))}}{1-e^{-2\pi y}}\mathrm{cm}^2$ $x = \frac{E}{\mathrm{Ry}}$ $y = \frac{1}{\sqrt{x-1}}$	Absorption cross section of hydrogen for photons with energy E.[b]

Table 1—Continued

Symbol/Value	Explanation
$\sigma_{\mathrm{H}\,m} = \sigma_{\mathrm{H}}(E_m)$	Absorption cross section of hydrogen for photons with energy E_m, for some m.
$\sigma_{\mathrm{He}\,m} = \sigma_{\mathrm{H}\,m} \times \left(6.53\frac{E_m}{E_1} - 0.22\right)$	Absorption cross section of helium for photons with energy E_m, for some m.[c]
$\alpha_{\mathrm{H2}} = 2.59 \times 10^{-13}\frac{cm^3}{s}$	Recombination coeff. for H to all excited levels.[a]
$\alpha_{\mathrm{He2}} = 2.73 \times 10^{-13}\frac{cm^3}{s}$	Recombination coeff. for He to all excited levels.[a]
$\alpha_{\mathrm{He2}\,^1P} = \frac{2}{3}\left(\frac{1}{4}\alpha_{\mathrm{He2}}\right)$	Recombination coeff. for He to excited singlet levels which cascade to the 2^1P state.
$\alpha_{\mathrm{He2}\,^1S} = \frac{1}{3}\left(\frac{1}{4}\alpha_{\mathrm{He2}}\right)$	Recombination coeff. for He to excited singlet levels which cascade to the $2\,^1S$ state.
$\alpha_{\mathrm{He2}\,^3P} = \frac{3}{4}\alpha_{\mathrm{He2}}$	Recombination coeff. for He to all excited triplet levels (all of which lead to the $2\,^3P$ state).
$\alpha_{\mathrm{He1}} = 1.59 \times 10^{-13}\frac{cm^3}{s}$	Recombination coeff. for He to ground.[a]
$\Re = n_\nu/\tau_\nu$	Rate of production of primary photons.
$\delta = \frac{E_0}{E_1} - 1$	Dimensionless excess energy of ionization photon.
$\phi = \frac{T_m}{T_\gamma}$	Ratio of the matter temperature to the radiation temperature.
$z' = 1 + z$	Inverse of the cosmic scale factor.
$\tau_{24} = \tau_\nu/10^{24}\mathrm{sec}$	Neutrino decay rate in units of 10^{24} seconds.

[a]From Osterbrock 1989 for $T = 10000K$.

[b]From Spitzer 1978.

[c]From Brown 1971.

Table 2.   Energy Values

Sym.	Value	Explanation
E_0	$\cdots$	Primary photon energy
E_1	24.6 eV[a]	Ionization energy of He
E_2	19.8 eV[a]	Energy of He $2\,^3P$
E_3	21.2 eV[a]	Energy of He $2\,^1P$
E_4	20.6 eV[a]	Energy of He $2\,^1S$
E_5	16.1 eV[b]	Avg. energy of γ absorbed by H in 2γ decay of He $2\,^1S$
Ry	13.6 eV	H ionization energy
ΔE	$E_0 - \mathrm{Ry}$	Energy deposited in a direct H ionization
ΔE_{He}	$E_0 - E_1$	Energy deposited in a He ionization

[a]From Osterbrock 1989.

[b]See Appendix C.